Semester	**THREE**
Course	**Major**
Paper Code	**C2ST230311T**
Paper Title	**Real Analysis I and Linear Algebra I**
No. of Credits	**4**
Theory / Practical /Composite	**Theory**
Classes per week	**4**
Module	**2**

Course Outcomes for Module I

1. Remember the definitions of sequences, series, limits, continuity, and differentiability, as well as the Archimedean property of real numbers.
2. **Understand** the properties of bounded and monotone sequences, the concept of absolute and conditional convergence, and the intermediate value property of continuous functions.
3. **Apply** various convergence tests—such as Ratio, Root, and Rabbe's tests—and apply Lagrange's Mean Value Theorem to solve mathematical problems.
4. **Analyze** the behavior of real-valued functions by examining left-hand and right-hand limits, infinite limits, and points of discontinuity.
5. Evaluate the convergence or divergence of real sequences and series using properties like the Cauchy criterion and specific convergence tests.
6. Create rigorous mathematical arguments to verify the properties of real-valued functions using sequential definitions of limits and continuity.

Course Outcomes for Module II

1. Remember the definitions and properties of special matrices (such as idempotent, orthogonal, and symmetric) and different types of vectors.
2. **Understand** the algebraic operations of matrices and determinants, including the concepts of linear independence and orthogonality.
3. **Apply** matrix transformations, partitioning of matrices, and Jacobi's Theorem to evaluate complex determinants and find matrix inverses.
4. **Analyze** the structure of vector sets by examining linear combinations and the relationship between linearly independent and orthogonal vectors.
5. Evaluate the properties of singular and non-singular matrices, as well as the trace and adjoint of various matrix forms.
6. Create orthonormal sets of vectors from a given basis by utilizing the Gram-Schmidt orthogonalization process.

Syllabus

Unit/Module	Content	No. of lectures	CO mapping	Cognitive levels
Module I				
Unit 1	***Real number system***: Basic Ideas. Archimedean property.	3	CO1	K1
	Sequences of real numbers: Definition, convergence, limit of a sequence. Bounded and monotone sequences. Cauchy sequences. Properties and applications.	7	CO1 CO2 CO5	K1 K2 K5
	Series of real numbers: Definition, convergence. Tests of convergence (statement and applications) – Comparison, Limit comparison, Ratio, Root, Condensation, Logarithmic, Integral tests, Abel's and Dirichlet's tests. Absolute and conditional convergence of series.	4	CO1 CO2 CO3 CO5	K1 K2 K3 K5
Unit 2	***Limits of real valued functions***: Definition, left hand and right hand limits. Infinite limits and limits at infinity. Sequential definition of limits. Properties of limits. Applications.	4	CO1 CO4 CO6	K1 K4 K6
	Continuity of real valued functions: Definition, left hand and right hand continuity. Discontinuous functions. Sequential definition of continuity. Properties of continuous functions. Applications. Intermediate value property.	4	CO1 CO4 CO6	K1 K4 K6
	Differentiability of real valued functions: Definition, properties. Chain rule. Rolle's theorem, Lagrange's mean value theorem. Applications.	4	CO2 CO3	K2 K3

Module II				
Unit 1	***Algebra of Matrices:*** A review - properties related to triangular, symmetric and skew-symmetric matrices, idempotent matrices, orthogonal matrices, singular and non-singular matrices and their properties. Trace of a matrix. Adjoint and inverse of a matrix and related properties. Partitioning of matrices and simple properties.	8	CO1 CO2 CO3 CO5	K1 K2 K3 K5
Unit 2	***Determinants:*** Definition, properties, algebraic operations and evaluation of determinants using transformations. Product of determinants. Vandermonde determinants for nth order. Jacobi's Theorem.	12	CO2 CO3	K2 K3
Unit 3	***Ideas of vectors:*** Definition, unit vector, null vector, sum vector, linear combination of vectors, linearly independent vectors, orthogonal vectors, orthonormal vectors. Gram-Schimdt orthogonalization.	6	CO1 CO2 CO4 CO6	K1 K2 K4 K6

Reading/Reference list

1.	Bertle R. G., Sherbert D. R. (2011): Introduction to Real Analysis, 4th Edition, Wiley & Sons Inc.
2.	Goldberg R. R. (2020): Methods of Real Analysis, Oxford & IBH Publishing Co Pvt Ltd.
3.	Ghorpade S. R., Limaye V. B. (2006): A Course in Calculus and Real Analysis, Springer Publications.
4.	Khuri A. (2003): Advanced Calculus with Applications in Statistics, 2nd Edition, Wiley Interscience.
5.	Rudin W. (2017): Principles of Mathematical Analysis, 3rd Edition, McGraw Hill Publication.
6.	Hadley G. (2002): Linear Algebra. Narosa Publishing House (Reprint).
7.	Mapa S. K. (2016): Higher Algebra: Abstract and Linear. Levant Books.
8.	Narayan S. (2004): A Textbook of Matrices, S Chand & Co Ltd.
9.	Searle S. R. (1982): Matrix Algebra Useful for Statistics. John Wiley & Sons.

Evaluation

<table>
<tr><td></td><td colspan="4">CIA: 30
Semester Exam: 70
Total: 100</td></tr>
<tr><td rowspan="3">Paper Structure for Theory Semester Exam</td><td colspan="2">Module-I (35 marks)</td><td colspan="2">Module-II (35 marks)</td></tr>
<tr><td>5 marks Questions</td><td>15 marks Questions</td><td>5 marks Questions</td><td>15 marks Questions</td></tr>
<tr><td>4 out of 6</td><td>1 out of 2</td><td>4 out of 6</td><td>1 out of 2</td></tr>
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CO	CO Description for Module I	Cognitive levels
CO1	**Remember** the definitions of sequences, series, limits, continuity, and differentiability, as well as the Archimedean property of real numbers.	K1
CO2	**Understand** the properties of bounded and monotone sequences, the concept of absolute and conditional convergence, and the intermediate value property of continuous functions.	K2
CO3	**Apply** various convergence tests—such as Ratio, Root, and Rabbe's tests—and apply Lagrange's Mean Value Theorem to solve mathematical problems.	K3
CO4	**Analyze** the behavior of real-valued functions by examining left-hand and right-hand limits, infinite limits, and points of discontinuity.	K4
CO5	**Evaluate** the convergence or divergence of real sequences and series using properties like the Cauchy criterion and specific convergence tests.	K5
CO6	**Create** rigorous mathematical arguments to verify the properties of real-valued functions using sequential definitions of limits and continuity.	K6

CO	CO Description for Module II	Cognitive levels
CO1	**Remember** the definitions and properties of special matrices (such as idempotent, orthogonal, and symmetric**)** and different types of vectors.	K1
CO2	**Understand** the algebraic operations of matrices and determinants, including the concepts of linear independence and orthogonality.	K2
CO3	**Apply** matrix transformations, partitioning of matrices, and Jacobi's Theorem to evaluate complex determinants and find matrix inverses.	K3
CO4	**Analyze** the structure of vector sets by examining linear combinations and the relationship between linearly independent and orthogonal vectors.	K4
CO5	**Evaluate** the properties of singular and non-singular matrices, as well as the trace and adjoint of various matrix forms.	K5
CO6	**Create** orthonormal sets of vectors from a given basis by utilizing the Gram-Schmidt orthogonalization process.	K6