Syllabus template

Semester: 8	
Programme: Mathematics	
Course: Differential Equations -3	
Paper Code:C4MT230852T & C4MT230852P	**Credits:**
Hours/week: 3 (Th.) +2(Pr.)	
Category: Core/MDC/SEC/VAC: Core (Elective)	
Theory / Practical / composite: Composite	
No of Modules: Nil	

Course Overview: Differential Equations-3

C4MT230852T(Theory)

This course develops analytical methods for classical partial differential equations and integral transforms arising in vibration, heat flow, and equilibrium problems. The PDE component derives the one-dimensional wave equation, obtains its general and d'Alembert solutions, and examines uniqueness, well-posedness, and initial / initial-boundary value problems. It covers infinite, semi-infinite, and finite strings with fixed/free ends, homogeneous and nonhomogeneous wave equations, and separation of variables with Fourier and non-Fourier representations. Heat conduction in rods and solids is derived and solved for finite rods under homogeneous and time-independent/time-dependent inhomogeneous boundary conditions, with uniqueness results. The Laplace equation is introduced with first, second, and third boundary conditions, followed by separation-of-variables treatment of Dirichlet boundary value problems. The integral-transform component builds the Fourier transform through definition, properties, transforms of derivatives, inversion, convolution and Parseval's theorem, and selected PDE applications. It then develops the Laplace transform, including elementary properties, transforms of derivatives, inverse transforms, convolution theorem, and applications to ordinary and partial differential equations. Throughout, emphasis is placed on derivation, analytical technique, correct use of boundary/initial data, verification of uniqueness, and interpretation of physically meaningful solutions. Learners completing the course should be able to model standard continuum phenomena, solve basic BVP/IBVP/IVPs rigorously, and apply transform methods effectively in mathematical and applied contexts.

C4MT230852P (Practical)

This course develops the practical and theoretical foundations of numerical methods for solving ordinary differential equations (ODEs), which arise whenever continuous change must be modeled—population dynamics, circuit analysis, chemical kinetics, mechanical vibrations, and projectile motion. Since most real-world ODEs resist closed-form solution, students learn to approximate solutions reliably using classical single-step, multistep, and finite difference techniques, while critically analyzing the accuracy, stability, and computational cost of each method.

C4MT230852T

Course Outcome: On successful completion of the course a student will be able to do the following:
1.Derive the one-dimensional wave equation governing string vibrations from physical postulates, and formulate initial and boundary value problems (Dirichlet, Neumann, mixed) with appropriate conditions.
2. Solve initial value problems for the infinite string (D'Alembert's solution) and initial boundary value problems for semi-infinite and finite strings (fixed/free ends), both via Fourier series and by direct methods**.**
3. Examine the uniqueness and well-posedness (existence, uniqueness, continuous dependence on data) of solutions to homogeneous and nonhomogeneous wave equations, and assess the effect of nonhomogeneous boundary conditions.
4. Solve the heat conduction equation in a finite rod using the method of separation of variables, including time-independent and time-dependent nonhomogeneous boundary conditions, and justify the uniqueness of the solution.
5. Apply the method of separation of variables to solve Dirichlet boundary value problems for the Laplace equation in standard geometries, and differentiate among first, second, and third kind boundary conditions.
6. Define the Fourier and Laplace transforms, state their elementary properties (linearity, shifting, scaling, transform of derivatives), and evaluate transforms and inverse transforms of commonly occurring functions.
7. Utilize the Convolution theorem and Parseval's identity to evaluate integrals, solve convolution-type equations, and deduce properties of transformed functions such as continuity and differentiability.
8. Construct solutions to ordinary and partial differential equations arising from physical problems (wave propagation, heat conduction) by selecting the appropriate integral transform technique, and evaluate the suitability of each transform for a given problem class.
C4MT230852P **Course Outcome:** On successful completion of the course a student will be able to do the following:
1.Define the fundamental concepts of single-step and multistep numerical methods for ordinary differential equations, including order of accuracy, local truncation error, and stability
2. Explain the geometric and algebraic rationale behind Euler's method and Heun's method as tangent-line and slope-averaging approximations.
3. Apply Euler's method, Heun's method, and Iterative (Modified) Euler's method to solve first-order initial value problems through structured tabular hand computations.
4. Apply the fourth-order Runge–Kutta (RK4) formula to compute accurate solutions of IVPs and generate startup values required by multistep schemes.
5. Construct solutions of IVPs using the explicit Adams–Bashforth multistep method, correctly organizing previous solution points for polynomial extrapolation.
6. Solve IVPs using the Adams–Bashforth–Moulton and Milne's predictor–corrector schemes, iterating the corrector to convergence.
7. Convert two-point boundary value problems into tridiagonal algebraic systems via the finite difference method and determine discrete solutions incorporating boundary conditions.

8. Analyze the truncation errors and convergence behavior of each method, and compare explicit versus implicit (predictor–corrector) strategies for equal step sizes.				
9. Evaluate the suitability and reliability of a chosen method (single-step, multistep, or finite difference) for a given ODE model, justifying the choice with computational and error evidence.				
10. Design a comparative computational study in which one representative ODE is solved by all seven methods, and present error-versus-step-size behavior graphically.				
Prerequisites: Differential and integral calculus; ordinary differential equations (including Sturm–Liouville theory); Fourier series; linear algebra; basic real analysis and complex variables. Multivariable calculus; elementary ordinary differential equations; Taylor series; numerical differentiation and integration.				
SYLLABUS				
UNIT/Module	**CONTENT**	**NO OF CLASSES**	**CO Mapping**	**COGNITIVE LEVEL**
I.	**C4MT230852T** **Wave Equation**: Derivation of 1dimention vibration and its general Solution. D'Alembert solution of Wave equation in an infinite string with the initial condition, Uniqueness of the solution, well posedness. Initial Boundary-Value Problems: Semi-infinite String with a Fixed End and with a Free End: Finite String with Fixed Ends and with one end free: solution with and without using Fourier Series. Nonhomogeneous Wave Equations: Solution and Uniqueness of the Solution: For Infinite String and finite string. Method of separation of variables to solve Finite String vibration with Fixed Ends and Free end.	**10**	**CO1, CO2, CO3**	**K3, K4, K5**
II.	**Conduction of Heat equation** in a rod and in Solids: Derivation, Solution in a finite Rod: Solution With time independent non-homogeneous boundary condition and with time dependent boundary condition: Uniqueness of the solution of heat conduction equation (Inhomogeneous) [5] **Laplace equation** Derivation and idea of 1^{st} 2^{nd} 3^{rd} boundary BVP.	**10**	**CO4, CO5**	**K3, K4**

	Solution of the Dirichlet BVP. [5] (solution of heat and Laplace equation use only method of separation of variables).			
III.	**Integral Transform:(Idea and applications) [20]** **Fourier transform**: Definition and elementary properties, continuity and differentiability. Fourier transform of derivative. Evaluation of some commonly occurring functions. Inverse Fourier transform, Convolution theorem and Parseval's theorem: some examples and application. Application to partial differential equation.[10] **Laplace Transform**: Definition and elementary properties, Laplace transform of derivatives. Evaluation of some elementary functions. Inverse Laplace transform, Convolution theorem: some examples and application. Application to ordinary and partial differential equation.	**20**	**CO6, CO7, CO8**	**K1, K2, K3, K4, K5, K6**
	C4MT230852P: **Solution of ODE:** 1. Euler and Heun Method 2. Iterative Euler's 3. Runge Kutta method of order 4 4. Adams Bashforth 5. Adams Bashforth Moulton method 6. Milnes method 7. Finite Difference method	**12**	**CO1, CO2, CO3, CO4, CO5, CO6, CO7, CO8, CO9, CO10**	**K1, K2, K3, K4, K5, K6**
Text Books				

1. Linear Partial Differential Equation for scientist and Engineers: Tyn Myint U, Lokenath Debnath.
2. Integral Transform and Their application: Lokenath Debnath and Dambaru Bhatta.

Suggested readings

1. An elementary Course in Partial differential Equation: T. Amarnath.
2. Integral Transform and Their application: Brian Devis.

Web Resources and eLearning Sources: MIT OCW 18.303 + NPTEL PDE, Paul's Online Notes + Dr. Trefor Bazett, MIT OCW 18.303 + LibreTexts, NPTEL Transform Techniques + MIT OCW 18.03 + 3Blue1Brown, Google Colab (Python).

Evaluation: Theory CIA: **10+5** Semester Exam: **45; Practical: CIA [38 +2]**

Paper Structure for Theory Semester Exam Module: 4 questions each of **10** marks out of a set of **7** questions and **1** question carrying **5** marks out of **2** questions.

Course outcomes (COs) and Cognitive Level Mapping

C4MT230852T

COs	CO Description	Cognitive levels
CO1	**Derive** the one-dimensional wave equation governing string vibrations from physical postulates, and formulate initial and boundary value problems (Dirichlet, Neumann, mixed) with appropriate conditions.	**K3, K4**
CO2	**Solve** initial value problems for the infinite string (D'Alembert's solution) and initial boundary value problems for semi-infinite and finite strings (fixed/free ends), both via Fourier series and by direct methods**.**	**K3**
CO3	**Examine** the uniqueness and well-posedness (existence, uniqueness, continuous dependence on data) of solutions to homogeneous and nonhomogeneous wave equations, and assess the effect of nonhomogeneous boundary conditions.	**K4, K5**
CO4	**Solve** the heat conduction equation in a finite rod using the method of separation of variables, including time-independent and time-dependent nonhomogeneous boundary conditions, and justify the uniqueness of the solution.	**K3, K4**
CO5	**Apply** the method of separation of variables to solve Dirichlet boundary value problems for the Laplace	**K3, K4**

	equation in standard geometries, and differentiate among first, second, and third kind boundary conditions.	
CO6	**Define** the Fourier and Laplace transforms, state their elementary properties (linearity, shifting, scaling, transform of derivatives), and evaluate transforms and inverse transforms of commonly occurring functions.	**K1, K2, K3**
CO7	**Utilize** the Convolution theorem and Parseval's identity to evaluate integrals, solve convolution-type equations, and deduce properties of transformed functions such as continuity and differentiability.	**K3, K4**
CO8	**Construct** solutions to ordinary and partial differential equations arising from physical problems (wave propagation, heat conduction) by selecting the appropriate integral transform technique, and evaluate the suitability of each transform for a given problem class.	**K5, K6**

Course outcomes (COs) and Cognitive Level Mapping

C4MT230852P

COs	CO Description	Cognitive levels
CO1	**Define** the fundamental concepts of single-step and multistep numerical methods for ordinary differential equations, including order of accuracy, local truncation error, and stability	**K1**
CO2	**Explain** the geometric and algebraic rationale behind Euler's method and Heun's method as tangent-line and slope-averaging approximations.	**K2**
CO3	**Apply** Euler's method, Heun's method, and Iterative (Modified) Euler's method to solve first-order initial value problems through structured tabular hand computations.	**K3**
CO4	**Apply** the fourth-order Runge–Kutta (RK4) formula to compute accurate solutions of IVPs and generate startup values required by multistep schemes.	**K3**
CO5	**Construct** solutions of IVPs using the explicit Adams–Bashforth multistep method, correctly organizing previous solution points for polynomial extrapolation.	**K3**

CO6	**Solve** IVPs using the Adams–Bashforth–Moulton and Milne's predictor–corrector schemes, iterating the corrector to convergence.	**K3**
CO7	**Convert** two-point boundary value problems into tridiagonal algebraic systems via the finite difference method and determine discrete solutions incorporating boundary conditions.	**K3**
CO8	**Analyze** the truncation errors and convergence behavior of each method, and compare explicit versus implicit (predictor–corrector) strategies for equal step sizes.	**K4**
CO9	**Evaluate** the suitability and reliability of a chosen method (single-step, multistep, or finite difference) for a given ODE model, justifying the choice with computational and error evidence.	**K5**
CO10	**Design** a comparative computational study in which one representative ODE is solved by all seven methods, and present error-versus-step-size behavior graphically.	**K6**