Semester	**ONE**
Course	**Major (Paper 2)**
Paper Code	**C1DS250121T**
Paper Title	**Real Analysis and Linear Algebra 1**
No. of Credits	**4**
Theory/Composite / Practical	**Theory**
Minimum No. of preparatory hours per week a student has to devote	**Module 1: 2 periods** **Module 2: 2 periods**
Number of Module	**TWO**
Syllabus	**Module-I** **Unit 1: Sets, Logic, and Relations** [10L] Sets, operations on sets, Cartesian product, power set, Propositional logic, predicates, quantifiers, Equivalence relations, partial orders, injective, surjective, and bijective functions, Cardinality & Infinite Sets-Cardinality of finite and infinite sets, Comparison of cardinalities, Cantor-Schroeder-Bernstein theorem (statement only). **Unit 2-Real Number System** [4L] Real number system: Basic ideas, Archimedean Property, completeness. **Unit 3- Sequences and Series of Real Numbers** [12L] Sequences: Definition, convergence, bounded and monotone sequences, Cauchy sequences. Series: Definition, absolute and conditional convergence. Tests of convergence (statement and applications): Comparison, Limit Comparison, Ratio, Root, Rabbe's, Cauchy Condensation, Logarithmic, Integral tests, Abel's and Dirichlet's tests. **Module-II** **Unit 1: Algebra of Matrices and Determinants:** Matrix representation of data tables and its relevance in data science. A review of matrix operations- addition, multiplication, transposition. Introduction to special classes of matrices- symmetric, skew-symmetric, orthogonal, and idempotent matrices. Concepts of trace, rank, and partitioning of matrices. Determinants: Definition, interpretation, and key properties. Determinant of product of matrices, effect of elementary row transformations on the determinant. Singular and non-singular matrices and their properties. Inverse of a matrix and related properties. Adjoint and cofactor. **[6]**

	Unit 2: Ideas of Vectors-Definition of a vector. Vector addition and scalar multiplication. Scalar product. Linear combination of vectors and the geometric interpretation of vector operations. Linear independence and dependence of a set of vectors. Orthogonal and orthonormal vectors. Gram-Schmidt orthogonalization process. **[6]** **Unit 3: Vector Space-**Vector spaces over the field of real numbers. Subspaces. Sum and intersection of subspaces. Span of a set of vectors. Basis and dimension. Projection. Orthogonal subspaces. Ortho-complement of a subspace. Row space and column space of a matrix. Null space and nullity of a matrix; connections with solvability of linear systems. **[10]** **Unit 4: Matrix Rank and Its Applications:** Definition of rank of a matrix, row rank, and column rank; equivalence of row rank and column rank. Standard theorems involving rank, including the Rank-Nullity Theorem. Rank of the sum and product of matrices. **[4]**
Learning Outcomes	• Applying fundamental concepts of real analysis to analyze the behavior of numerical methods and optimization techniques used in data science. • Evaluating the convergence of sequences and series using standard mathematical tests. • Utilizing set theory, logic, and relations to model data structures, relational databases, and logical reasoning in artificial intelligence. • Recalling the fundamental concepts of combinatorics and Venn diagrams. • Understanding matrix operations, special matrix types, and determinants to interpret and manipulate structured data in tabular (matrix) form. • Applying vector operations and vector space concepts to analyze multidimensional data structures relevant to data science. • Analyzing matrix rank and subspace relationships to assess the solvability of linear systems and to identify dimensionality and dependencies in datasets.
Reading/Reference List	1. Bertle R. G., Sherbert D. R. (2011): Introduction to Real Analysis, 4th Edition, Wiley & Sons Inc. 2. Goldberg R. R. (2020): Methods of Real Analysis, Oxford & IBH Publishing Co Pvt Ltd. 3. Khuri A. (2003): Advanced Calculus with Applications in Statistics, 2nd Edition, Wiley Interscience. 4. Rudin W. (2017): Principles of Mathematical Analysis, 3rd Edition, McGraw Hill Publication. 5. Rosen, K. H. (2019): Discrete Mathematics and Its Applications, 8th Edition, McGraw-Hill. 6. Kolman, B., Busby, R., Ross, S. C. (2013): Discrete Mathematical Structures, 6th Edition, Pearson. 7. Hadley, G. (2002): Linear Algebra. Narosa Publishing House (Reprint). 8. Mapa, S. K. (2016): Higher Algebra: Abstract and Linear. Levant Books. 9. Narayan, S. (2004): A Textbook of Matrices. S. Chand & Co. Ltd. 10. Searle, S. R. (1982): Matrix Algebra Useful for Statistics. John Wiley & Sons. 11. Lay, D. C., Lay, S. R., & McDonald, J. J. (2015): Linear Algebra and Its Applications (5th Edition). Pearson.
Evaluation	CIA: 30 Semester exam: 70

	Total: 100	
Paper Structure for Theory Semester Exam	Module-I (35 marks)	Module-II (35 marks)
	To answer Short: 4 out of 6 (5 marks) Long: 1 out of 2 (15 marks)	To answer Short: 4 out of 6 (5 marks) Long: 1 out of 2 (15 marks)