Syllabus template

Semester: 7	
Programme: Mathematics	
Course: Algebra-3 & Ordinary Differential Equations-1	
Paper code: B4MT230711T	**Credits:4**
Hours/week: 4	
Category: Core/MDC/SEC/VAC: Minor	
Theory / Practical / Composite: Theory	
No of Modules: Nil	

Course Overview: Algebra-3 & Ordinary Differential Equations-1

This course integrates foundational concepts of linear algebra and ordinary differential equations to build analytical and problem-solving skills. It begins with vector spaces over $\mathbb{R}$, covering linear dependence, basis, dimension, and linear operators with matrix representations. The course develops inner product spaces, including orthogonality, Gram–Schmidt process, and key inequalities, followed by eigenvalues, eigenvectors, diagonalization, and applications of the Cayley–Hamilton theorem. Real quadratic forms are also introduced. The differential equations component focuses on the formulation and solution of first-order equations, including exact and non-exact equations, linear and Bernoulli equations, and special types such as Clairaut's equation. Emphasis is placed on methods of solution and applications across disciplines. Overall, the course provides a strong foundation in algebraic structures and differential equations essential for advanced studies in mathematics and applied sciences.

Course Outcome: On successful completion of the course a student will be able to do the following:

1. **Explain** the structure of vector spaces over $\mathbb{R}$, including subspaces, linear dependence, span, and basis.
2. **Apply** concepts of linear operators and represent them using matrices in finite-dimensional vector spaces.
3. **Apply** inner product space concepts, including orthogonality and Gram–Schmidt process, to construct orthonormal bases and solve related problems.
4. **Analyze** eigenvalues and eigenvectors, and perform diagonalization of matrices using appropriate methods, including applications of Cayley–Hamilton theorem.
5. **Solve** problems involving real quadratic forms and interpret their algebraic and geometric significance.
6. **Formulate and solve** first-order ordinary differential equations, including exact, non-exact, linear, Bernoulli, and higher-degree equations.
7. **Analyze** special types of differential equations such as Clairaut's equation and interpret general and singular solutions.

Prerequisites:

SYLLABUS

UNIT/Module	CONTENT	NUMBER OF CLASSES	CO Mapping	COGNITIVE LEVEL
I.	Algebra-3: Linear Spaces and Linear Operators [16]: Definition of vector space over R: examples [2], concepts of linear span, linear dependence and independence of a finite set of vectors, subspaces [4], Idea of basis of a finite dimensional real vector spaces[2]. Statement of addition, deletion and replacement theorems: examples[2]. Linear Operators on linear spaces: matrix representation [6]. Inner Product Spaces [6]:Definition and examples, Orthogonal and Orthonormal set of vectors[2], Schwarz Inequality, Gram-Schimdt process [4]. Eigenvalues & Diagonalization of Matrices[11]:Characteristic equation of a square matrix, eigen values and eigenvectors and related problems[5]. Statement of Cayley Hamilton theorem and its applications[2]. Diagonalization of Matrices[4]. Real Quadratic Forms [3].	**36 classes**	**CO1, CO2, CO3, CO4, CO5**	**K2, K3, K4**
II.	Ordinary Differential Equations-1: Formation of ode -exemplification from various fields (3). First order ode :Exact differential equations, Non-exact differential equations & Integrating factors (no proof) (6). Linear ode and Bernoulli's equation (2), Differential Equations of first order and higher degree[3]; Clairaut's equation: general& singular solutions [2].	**16 classes**	**CO6, CO7**	**K3, K4**
Text Books				
1. Higher Algebra (Abstract & Linear): S.K.Mapa.				
2. Linear Algebra and its applications: Gilbert Strang.				
3. Differential Equations: Ghosh & Chakraborty.				

Suggested readings
1. Linear Algebra: Hoffman Kunze.
2. Modern Higher Algebra: Galois Theory: Emil Artin.
3. Differential Equations: Shepley L. Ross.
Web Resources
Evaluation: Theory **CIA**: **20**+5+5=**30; Semester Exam**: **70**
Paper Structure for Theory Semester Exam Module: 7 questions each of **10** marks out of a set of **13** questions.

Course outcomes (COs) and Cognitive Level Mapping

COs	**CO Description**	**Cognitive levels**
CO1	Explain the structure of vector spaces over $\mathbb{R}$, including subspaces, linear dependence, span, and basis.	**K2**
CO2	Apply concepts of linear operators and represent them using matrices in finite-dimensional vector spaces.	**K3**
CO3	Apply inner product space concepts, including orthogonality and Gram–Schmidt process, to construct orthonormal bases and solve related problems.	**K3, K4**
CO4	Analyze eigenvalues and eigenvectors, and perform diagonalization of matrices using appropriate methods, including applications of Cayley–Hamilton theorem.	**K4**
CO5	Solve problems involving real quadratic forms and interpret their algebraic and geometric significance.	**K3, K4**
CO6	Formulate and solve first-order ordinary differential equations, including exact, non-exact, linear, Bernoulli, and higher-degree equations.	**K3**
CO7	Analyze special types of differential equations such as Clairaut's equation and interpret general and singular solutions.	**K4**