Syllabus template

Semester: IV	
Programme : MATHEMATICS	
Course: Algebra-3	
Paper code: C2MT230421T	**Credits: 4**
Hours/week : 4 HOURS	
Category: Core/MDC/SEC/VAC : CORE	
Theory / Practical / Composite : THEORY	
No. of Modules : NA	

Course Overview: This course focuses on advanced topics in linear algebra and linear transformations. The course explores rank–nullity theorem, row and column spaces, and ranks of matrices. It introduces dual spaces, dual bases, and annihilators, along with eigenvalues, eigenspaces, diagonalization, minimal polynomials, and canonical forms. Students study inner product spaces, orthogonality, the Gram–Schmidt process, and Bessel's inequality. The course also covers adjoint, normal, and self-adjoint operators, providing a deeper understanding of the structure and properties of linear operators.

Course Outcome:

CO1: Apply linear transformations and the Rank–Nullity Theorem to solve problems involving matrices and vector spaces.

CO2: Explain and analyze the concepts of row space, column space, row rank, column rank, determinant rank, and determine the rank of the product of matrices.

CO3: Describe the concepts of dual spaces, dual bases, double duals, annihilators, and interpret the transpose of linear transformations with respect to dual bases.

CO4: Analyze eigenspaces, invariant subspaces, diagonalizability, the Cayley–Hamilton theorem, minimal polynomial, and canonical forms of linear operators.

CO5: Apply the concepts of inner product spaces including norms, orthogonal and orthonormal bases, the Gram–Schmidt orthogonalization process, orthogonal complements, and related inequalities.

CO6: Explain the adjoint of linear operators and analyze the properties and diagonalizability of normal and self-adjoint operators.

Prerequisites: Students should have a foundational understanding of undergraduate **linear algebra and abstract algebra**, including vector spaces, matrices, determinants, and systems of linear equations. Familiarity with linear transformations, basic properties of groups, and elementary proof techniques is desirable. Knowledge of eigenvalues, eigenvectors, and basic matrix operations will help students engage effectively with advanced topics such as dual spaces, inner product spaces, and diagonalization of linear operators covered in this course.

SYLLABUS:

UNIT/Module	CONTENT	HOURS or NUMBER OF CLASSES	CO Mapping	COGNITIVE LEVEL
I.	Solving problems of matrices by the use of linear transformations, the rank-nullity theorem (2).	**2 hours**	**CO1**	**K3,K4**
II.	Row space and column space of a matrix. Row rank, column rank, determinant rank and their	**14 hours**	**CO1.CO2,CO3**	**K2,K3,K4**

	equality. Rank of product of two matrices (6). Dual spaces, dual basis, double dual (4), transpose of a linear transformation and its matrix in the dual basis, annihilators (4).			
III.	Eigen spaces of a linear operator, diagonalizability, invariant subspaces and Cayley-Hamilton theorem (8), the minimal polynomial for a linear operator, diagonalizability in connection with minimal polynomial, and canonical forms (8).	**16 hours**	**CO4**	**K2,K4,K5,K6**
IV.	Inner product spaces and norms- Examples, Cauchy-Schwarz Inequality (4), Orthogonal and orthonormal basis, Gram-Schmidt orthogonalization process (5), orthogonal complements, Bessel's inequality (3).	**12 hours**	**CO5**	**K4,K5,K6**
V.	The adjoint of a linear operator. Normal and self-adjoint operators and their diagonalizability (8)	**8 hours**	**CO6**	**K4,K5**
Text Books				
1. Linear Algebra: Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence.				
2. Linear Algebra--a Geometric Approach: S. Kumaresan.				
3. Linear Algebra: Kenneth Hoffman, Ray Kunze				
Suggested readings				
1. Linear Algebra Done Right: Sheldon Axler.				
2. Higher Algebra (Linear and Abstract): S.K.Mapa.				
3. Introduction to Linear Algebra: Gilbert Strang.				
Web Resources				
1.				
2.				
3.				
4.				
Evaluation: Theory CIA: 20+5+5=30 Semester Exam: 70				
Paper Structure for Theory Semester Exam Module : 7 questions each carrying 10 marks out of 13/14 questions				

Course outcomes (COs) and Cognitive Level Mapping

COs	CO Description	Cognitive levels
CO1	Apply linear transformations and the Rank–Nullity Theorem to solve problems involving matrices and	K2.K3,K4

	vector spaces.	
CO2	Explain and analyze the concepts of row space, column space, row rank, column rank, determinant rank, and determine the rank of the product of matrices.	K2,K3,K4
CO3	Describe the concepts of dual spaces, dual bases, double duals, annihilators, and interpret the transpose of linear transformations with respect to dual bases.	K2,K3,K4
CO4	Analyze eigenspaces, invariant subspaces, diagonalizability, the Cayley–Hamilton theorem, minimal polynomial, and canonical forms of linear operators.	K2,K4,K5,K6
CO5	Apply the concepts of inner product spaces including norms, orthogonal and orthonormal bases, the Gram–Schmidt orthogonalization process, orthogonal complements, and related inequalities.	K4,K5,K6
CO6	Explain the adjoint of linear operators and analyze the properties and diagonalizability of normal and self-adjoint operators.	K4,K5