Syllabus template

Semester: 5	
Programme: Mathematics	
Course: Probability Theory	
Paper code: C3MT230511T	Credits: 4
Hours/week : 4 hours	
Category: Core/MDC/SEC/VAC : Core	
Theory / Practical / Composite : Theory	
No of Modules : Nil	

Course Overview: Probability Theory

This course provides a rigorous foundation in the mathematical principles of probability theory, emphasizing both its conceptual framework and analytical techniques. The course begins with an interpretation of randomness in non-deterministic experiments and introduces the construction of sample spaces—both countable and uncountable—arising from various random phenomena. Fundamental set-theoretic structures such as fields and σ-fields are explored thereby leading to the construction of minimal σ-fields and the introduction of Borel fields and Borel sets in $\mathbb{R}$. This course helps students to have a clear understanding of events and event spaces, including special types such as sure, impossible, mutually exclusive, and exhaustive events. The course also develops the concept of sequences of events—monotone, expanding, and contracting—and their limiting behaviour, including limit superior and limit inferior.

The classical Laplace definition of probability is introduced through combinatorial illustrations, followed by a critical discussion of its limitations. The axiomatic formulation of probability by Kolmogorov is then presented as a modern alternative, emphasizing the role of probability as a set function on both countable and uncountable sample spaces. Core properties of probability functions are derived, including the Addition Theorem, Boole's and Bonferroni's inequalities, and the Axiom of Continuity, with selected proofs grounded in mathematical induction.

Building on these foundations, the course advances to conditional probability, the multiplication rule, the law of total probability, and Bayes' theorem, illustrating each through real-world problem-solving. The concept of independence among events—pairwise and mutual—is developed for both finite and countably infinite collections.

A major focus of the latter part of the course is the introduction of random variables as measurable functions. Students will explore the construction of induced probability spaces, distribution functions, and their properties derived from the axioms of probability. The course distinguishes between discrete, continuous, and absolutely continuous random variables, developing associated probability mass functions (pmf) and probability density functions (pdf). Transformations of random variables (both one- and two-dimensional) are treated analytically, with applications to common distributions such as Binomial, Poisson, Uniform, and Normal.

Moments of distributions—raw and central—are analysed with interpretations of mean, variance, skewness, and kurtosis. In the multivariate setting, the course introduces joint,

marginal, and conditional distributions, bivariate Normal and Uniform models, and explores the concepts of statistical dependence and independence at the distributional level. Covariance, correlation, and their interpretations are studied in depth, culminating in the derivation of the linear regression equation using the principle of least squares.
Course Outcome: Probability Theory
1. Understand the concept of randomness, sample spaces (countable and uncountable), and events, including sigma-fields, Borel sets, and event operations in probabilistic frameworks.
2. Apply the axiomatic and classical definitions of probability to construct probability spaces, derive basic properties of probability measures, and prove related theorems such as the addition theorem, Boole's and Bonferroni's inequalities, and interprete the axiom of continuity.
3. Analyze and compute conditional probabilities, interpret independence of events, and apply theorem of total probability and Bayes' theorem to solve real-world and theoretical problems.
4. Construct one-dimensional random variables as measurable functions on probability spaces, and obtain their distribution functions and analyze their properties using the measure-theoretic foundation of probability.
5. Classify types of random variables (discrete, continuous, absolutely continuous) through their pmf, pdf, and cdf; and apply transformations for one-dimensional random variables in both discrete and continuous cases.
6. Determine important probability distributions such as Binomial, Poisson, Uniform, and Normal, and evaluate their moments, including mean, variance, skewness, and kurtosis, for statistical interpretation.
7. Formulate and analyze two-dimensional random variables, their joint, marginal, and conditional distributions, and solve problems involving bivariate uniform and normal distributions.
8. Interpret statistical dependence and independence between random variables using joint pdf/pmf concepts, and evaluate covariance, correlation coefficient, and linear regression equations using least squares.
Prerequisites: *Basic knowledge about any prior course: Permutation and Combination*
SYLLABUS: Probability Theory

UNIT/Module	CONTENT	HOURS or NUMBER OF CLASSES	CO Mapping	COGNITIVE LEVEL
I.	Interpretation of randomness in non-deterministic experiments. Illustration of countable and uncountable sample spaces connected to different random experiments. [1] Event and Event space: need of the ideas of field & sigma field in probability theory. Idea of minimal sigma-field. Construction of minimal sigma field. Borel field &Borel sets in R [3] Definitions of sure event ,impossible event ,	6 hours	CO1	K2

	mutually exclusive and exhaustive events along with examples. Monotone sequence of events: contracting and expanding sequence. Limit of monotone sequence of events. Limit superior and limit inferior of an arbitrary sequence of events.[2]			
II.	Introduction to the idea of probability: different types of definition: Classical Laplace definition: some simple combinatorial problems based on it [3]- Shortcomings of this classical approach and Axiomatic approach as an alternative. [Kolmogorov's probability axioms].[1] Probability defined as a set function on countable and uncountable sample spaces. Properties of probability function, Addition Theorem on Probability (proof based on Mathematical Induction included)[3].Concept of Probability space. Boole's and Bonferroni's inequalities. Axiom of Continuity [Proofs included] [3]	10 hours	CO2	K3
III.	Conditional Probability- definition , examples, verification that conditional probability is a probability in its own right.[1] Multiplication rule of probability, Theorem of total probability, Bayes' theorem and related problems. [3] Independence of two events- extension to a finite/ countably infinite collection of events, Pairwise and mutual independence, problems.[2]	6 hours	CO3	K4
IV.	Introduction to one dimensional random variables: Random variable as Borel measurable function. Induced probability space [its justification as probability space according to Kolmogorov's axioms] [2]. Distribution function and	14 hours	CO4, CO5, CO6	K3, K4, K5

	derivation of its properties from those of probability measure [3] Classification of random variables: discrete , continuous and absolutely continuous types based on the nature of the distribution function . Probability mass function (pmf) and probability density function (pdf)[1] . Transformation of one dimensional random variable (discrete and continuous) –theory and problems.[2] Examples of Discrete and Continuous random variables: Binomial distribution –Poisson approximation. Poission distribution ,Uniform distribution, Normal distribution and its symmetry features.[3] Raw and central moments for univariate distributions [1]. Mean, median, variance, skewness and kurtosis and their interpretations [2].			
V.	Two- dimensional random variable: definition and examples, probability space for the two -dimensional random variables: Joint distribution function. Marginal distribution function [3] Joint probability mass function and joint probability density function definition and simple problems related to them [2]. Bivariate Normal and Uniform distributions.[3] Transformation of two-dimensional continuous random variables and related problems[3] Statistical dependence and independence of two random variables- discussions at the pmf/pdf level [1] Moments for jointly distributed random variables. Covariance and correlation coefficient: properties &Interpretations.[2] Determination of linear regression equation using	16 hours	CO7, CO8	K4, K5, K6

	principles of least squares [2]			

Text Books
1. Mathematical Probability: Banerjee, De & Sen
2. Introduction to Probability Theory: Sheldon Ross
3.
Suggested readings
1. Basic Probability Theory: Robert B-Ash
2. Modern Probability Theory: B.R.Bhatt
3. Introduction to Probability Theory: Hoel, Port & Stone
Web Resources
1.
2.
3.
4.
Evaluation: End Sem;70 CIA:30 (20 (MidSem)+5(Assignment)
Paper Structure for Theory Semester Exam:: 7 questions each carrying 10 marks out of 13/14 questions.

Course outcomes (COs) and Cognitive Level Mapping

COs	CO Description	Cognitive levels
CO1	Understand the concept of randomness, sample spaces (countable and uncountable), and events, including sigma-fields, Borel sets, and event operations in probabilistic frameworks.	K2
CO2	Apply the axiomatic and classical definitions of probability to construct probability spaces, derive basic properties of probability measures, and prove related theorems such as the addition theorem, Boole's and Bonferroni's inequalities, and interpret the axiom of continuity	K3
CO3	Analyze and compute conditional probabilities, interprete independence of events, and apply theorem of total probability and Bayes' theorem to solve real-world and theoretical problems.	K4
CO4	Construct one-dimensional random variables as measurable functions on probability spaces, and obtain their distribution functions and analyze their properties using the measure-theoretic foundation of probability	K3, K4
CO5	Classify types of random variables (discrete, continuous, absolutely continuous) through their pmf, pdf, and cdf; and apply transformations for one-dimensional random variables in both discrete and	K3

	continuous cases.	
CO6	Determine important probability distributions such as Binomial, Poisson, Uniform, and Normal, and evaluate their moments, including mean, variance, skewness, and kurtosis, for statistical interpretation.	K4, K5
CO7	Formulate and analyze two-dimensional random variables, their joint, marginal, and conditional distributions, and solve problems involving bivariate uniform and normal distributions.	K4
CO8	Interpret statistical dependence and independence between random variables using joint pdf/pmf concepts, and evaluate covariance, correlation coefficient, and linear regression equations using least squares.	K5, K6