Syllabus template

Semester: 5	
Programme : Mathematics	
Course : Metric Spaces	
Paper code: C3MT230521T	**Credits: 4**
Hours/week : 4hrs	
Category: Core/MDC/SEC/VAC : Core	
Theory / Practical / Composite : Theory	
No of Modules : NA	

Course Overview:

The course **Metric Spaces** introduces fundamental concepts of metric space topology and analysis. It covers the definition and examples of metric spaces, open and closed sets, interior, closure, dense subsets, and subspaces. The course explores convergence of sequences, Cauchy sequences, and completeness in metric spaces. Key topics include continuity, homeomorphisms, connectedness, compactness, and uniform continuity. Important theorems such as Cantor's Intersection Theorem, Banach Fixed Point Theorem, and Heine–Borel Theorem are discussed. The course emphasizes understanding the topological structure of metric spaces and analyzing properties preserved under continuous and homeomorphic mappings

Course Outcome:

1. **Explain and apply** the fundamental concepts of **metric space topology** and analyze their basic properties and applications.
2. **Analyze** the concepts of **convergence of sequences**, **Cauchy sequences**, and **completeness** in metric spaces and apply them to solve mathematical problems
3. **Apply and evaluate** the concepts of **continuity** and **compactness** in metric spaces and examine their preservation under **homeomorphisms**.
4. **Analyze and interpret** the concept of **connectedness** in metric spaces and evaluate its preservation under **homeomorphic mappings**.

Prerequisites: ***Basic knowledge about Real Analysis.***

SYLLABUS

UNIT/Module	**CONTENT**	**HOURS or NUMBER OF CLASSES**	**CO Mapping**	**COGNITIVE LEVEL**
I.	Examples of metric on the set R , the two dimensional plane &three dimensional space .Definition and examples of abstract metric spaces: l discrete metric space, Sup- metric on C[a,b] , Minkowski's inequality. *l p* space. standard bounded metrics, Idea of Pseudo-metric . (2) Open and closed balls in various metric spaces, its geometries. Interior points, open sets and its examples and basic	**12hrs**	**CO1**	**K2,K3**

	properties (1), The topology of a metric space, structure of open sets in a metric space, intersection of infinite numbers of open sets may not be open (1). Interior of a set and its basic properties. Limit points, closed sets and its basic properties (1). Closure of a set and its basic properties in a metric space, union of infinite numbers of closed sets may not be closed. Closure of A is the smallest closed set containing A. Dense subsets of a metric space (1). Subspace of a metric space, structure of open and closed sets in subspace (1). Relations of interior and closure of a set in a subspace in comparison with that in the mother space (1). Bounded sets in a metric space. Diameter of a subset of a metric space and its properties (1). Concept of distance of a sub set from a point, examples, distance between two subsets in a metric spaces its examples and properties, two disjoint closed sub sets may have zero distance . Equivalent metric, its examples. Every metric space has an equivalent metric which is bounded (3).			
II.	Concept of convergence of a sequence in a metric space with examples, general definition, uniqueness of limit of a sequence in a metric space, bounded sequence, convergent sequence is bounded, bounded sequence may not have a convergent sub sequence (make a tally with the results in usual metric space) (2). Cauchy sequence, Cauchy sequence may not be convergent but it is bounded. Convergent sequence is Cauchy (2). The concept of completeness in a metric space with examples, definition and non-examples. A subspace in a complete metric space (M,ρ) is complete iff it is closed in (M,ρ) (3). R with the usual metric is complete and its other equivalent forms and their implications (2). Cantor's Intersection Theorem and its applications (2).	**11hrs**	**CO2**	**K1,K2,K4**
III.	Concept of continuity of functions between metric spaces. Continuity of real valued functions. Continuity at a point and continuity on a set.	**7hrs**	**CO3**	**K2,K3,K4**

	Sequential criteria of continuity. Continuity via open sets. Composition of continuous functions is again continuous, algebra of continuous functions. Continuity of vector valued functions (3). Homeomorphic and isometric spaces— homeomorphism is topological equivalence isometricism is metrical equivalence. Topological properties, completeness is not a topological property. Homeomorphic subsets of R.R 2 , R 3 (4).			
IV.	The concept of Connectedness in a metric space, connected sub sets of real line , some special connected subsets of plane and space (1): Intermediate Value Theorem and its applications. Continuous image of a connected subset of a metric space is connected, connectedness is a topological property. Intersection and union of connected space may not be connected, subspace of connected space may not be connected (1). Classification of subsets of reals, plane and space via connectedness (2) Product of two connected Space is connected (1)	**5hrs**	**CO4**	**K2,K3,K4**
V.	Bounded sequence may not have a convergent subsequence in a metric space, Concept of sequential compactness (1). Bounded infinite subset in a metric space may not have limit points, the concept of BW-Compactness. Equivalence of sequential compactness and BW-compactness (1). Compactness via open cover, Sub space of a compact space may not be compact, Intersection and union of compact spaces. Closed subset in a compact metric space is compact (2). Finite Intersection Property and its u se to prove the non-compactness of Euclidean spaces (2) Total boundedness, compactness vs completeness, Banach Fixed point theorem (2). Proofs of four equivalent statements, viz , (a) (M,ρ) is compact MS (b) (M,ρ) is sequentially compact MS (c) (M,ρ)is complete MS and totally bounded (d) (M,ρ) has BWP (2). Compact subsets of the set of reals, plane and space. Compact	**17hrs**	**CO3, CO1**	**K2.K3,K4,K1**

	subsets are closed and bounded in a metric space converse may not be true in general, Heine-Borel theorem , closed and bounded sets i.e. compact sets in R must have maximum/minimum elements. Continuous image of a compact set is compact. Compactness is a topological property. Real valued continuous function defined on compact domain is bounded and attains its maxima and minima (3). The concept of Uniform continuity of functions, examples, non-examples. Uniform continuous function takes Cauchy sequence into a Cauchy sequence, uniform continuous function has unique continuous extension on the closure of its domain. The distance function from a fixed set in a metric space is uniformly continuous, disjoint closed subsets have positive distance if one of them is compact. Continuous functions defined on compact domain is uniformly continuous (2). An open continuous bijection on a compact domain is homeomorphism. Classification of subsets of the set of R.R^2 , R^3via compactness (2).[24]			

Text Books

1. Topology of Metric Spaces — S. Kumaresan
2. Elements of Abstract Analysis—Micheal O Searcoid
3. Metric Spaces— P.K. Jain & Khalil Ahmed
4. Introduction to Topology and Modern Analysis— G. F. Simmon

Suggested readings

1. Metric Spaces— Victor Bryant
2. Introductory Real analysis— M. E. Munroe
3. Metric Spaces— Satish Shirali & H. L. Vasudeva

Web Resources

1.
2.
3.
4.

Evaluation : Theory CIA: 20+5+5=30 Semester Exam: 70

Paper Structure for Theory Semester Exam Module : 7 questions each carrying 10 marks out of 12/13 questions.

Course outcomes (COs) and Cognitive Level Mapping

COs	CO Description	Cognitive levels
CO1	**Explain and apply** the fundamental concepts of **metric space topology** and analyze their basic properties and applications.	**K1,K2,K3,K4**
CO2	**Analyze** the concepts of **convergence of sequences**, **Cauchy sequences**, and **completeness** in metric spaces and apply them to solve mathematical problems	**K1,K2,K4**
CO3	**Apply and evaluate** the concepts of **continuity** and **compactness** in metric spaces and examine their preservation under **homeomorphisms**.	**K2,K3,K4**
CO4	**Analyze and interpret** the concept of **connectedness** in metric spaces and evaluate its preservation under **homeomorphic mappings**.	**K2,K3,K4**