

## Syllabus template

| <b>Semester: 5</b>  |   |   |                       |                            |
|---|---|---|-----------------------|----------------------------|
| <b>Programme : Mathematics</b>  |   |   |                       |                            |
| <b>Course : Multivariate Calculus</b>   |   |   |                       |                            |
| <b>Paper code: C3MT230541T</b>  |   |   | <b>Credits:4</b>      |                            |
| <b>Hours/week : 4</b>   |   |   |                       |                            |
| <b>Category: Core/MDC/SEC/VAC : Core</b>  |   |   |                       |                            |
| <b>Theory / Practical / Composite : Theory</b>  |   |   |                       |                            |
| <b>No of Modules : Nil</b>  |   |   |                       |                            |
| <p><b>Course Overview:</b> This course builds on multivariable calculus and analysis concepts, extending one's understanding of one variable calculus to the functions of several variables. Key topics include limits and continuity for functions primarily of two real variables and vector-valued functions; Partial and directional derivatives, and their relationship with continuity. Equality of mixed partial derivatives. Differentiability of two-variable and vector-valued functions: linear and bilinear maps, Jacobian matrix, chain rule. The Mean Value Theorem for functions of two and three variables. The Inverse Function Theorem and Implicit Function Theorem with applications. Geometry of level surfaces: tangent planes, gradient as normal to the level surfaces, extreme values including constrained extrema (Lagrange multiplier method). Introduction to double integrals and applications such as computing area and volume.</p> |   |   |                       |                            |
| <b>Course Outcome:</b>  |   |   |                       |                            |
| 1. This course on Multivariate Calculus helps in understanding and applying the concepts of limit, continuity and differentiability for functions of more than one variable;  |   |   |                       |                            |
| 2. Recognize and interpret the derivative as a linear map and identify Jacobian as the derivative. Grasp the geometric role of the gradient of a function and its interpretation as a normal to the level surfaces.   |   |   |                       |                            |
| 3. Application of the chain rule, Jacobians and their roles in checking functional dependence of functions; Mean Value Theorem in 2D /3D and its geometrical interpretation;  |   |   |                       |                            |
| 4. Implicit Function Theorem and Inverse Function Theorem involving two and three variables in appropriate problems.  |   |   |                       |                            |
| 5. One can learn extreme values of multivariable functions (unconstrained & constrained) and execute their applications in various contexts.  |   |   |                       |                            |
| 6. Introduction to double integrals and applications such as computing area and volume is also an objective of the course.  |   |   |                       |                            |
| <b>Prerequisites:</b> Basic knowledge of One variable Calculus studied in previous semesters.   |   |   |                       |                            |
| <b>SYLLABUS</b>   |   |   |                       |                            |
| <b>UNIT/Module</b>  | <b>CONTENT</b>  | <b>HOURS<br/>or<br/>NUMBER<br/>OF<br/>CLASSES</b> | <b>CO<br/>Mapping</b> | <b>COGNITIVE<br/>LEVEL</b> |
| <b>I.</b>   | Limit, Continuity of functions of two real variables, continuity of vector valued | <b>12</b>   | <b>CO1</b>            | <b>K2,K4</b>               |

|             |   |           |            |              |
|-------------|---|-----------|------------|--------------|
|             | functions. Partial derivatives and directional derivatives and its relation with continuity: sufficient conditions of continuity.   |           |            |              |
| <b>II.</b>  | Differentiability of two variables functions and its relation with partial derivatives and continuity, sufficient conditions of differentiability, commutativity of mixed order partial derivatives. Homogeneous functions and Euler's Theorem for two & three variables. | <b>12</b> | <b>CO2</b> | <b>K2,K3</b> |
| <b>III.</b> | Differentiability of vector valued functions, examples, linear maps , bilinear maps, differentiability of standard maps, Jacobian matrix and chain rule .Application and properties of Jacobians  | <b>10</b> | <b>CO3</b> | <b>K3,K4</b> |
| <b>IV.</b>  | Mean value theorem of scalar valued maps on vector domains, functions of vanishing partial derivatives on connected domains. Inverse function theorem, Implicit Function theorem and their applications.  | <b>7</b>  | <b>CO4</b> | <b>K2,K3</b> |
| <b>V.</b>   | Critical points, Extreme values, Lagrange's multiplier method, local expression of a function near non-degenerate critical points . Critical points, Extreme values, Lagrange's multiplier method, local expression of a function near non-degenerate critical points     | <b>6</b>  | <b>CO5</b> | <b>K5</b>    |
| <b>VI.</b>  | Introduction to double integrals and applications such as computing area and volume.  | <b>5</b>  | <b>CO6</b> | <b>K3</b>    |

**Text Books**

1. G. B. Thomas / R. L. Finney – *Calculus and Analytic Geometry* (multivariate calculus, double integrals)
2. Advanced Calculus-An Introduction to Classical Analysis-Louis Brand[Dover, Inc]

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|---|
| 3. T.M.Apostol-Calculus {Vol-II}  |
| <b>Suggested Readings</b>   |
| 1. S. Mukherjee- Advanced Calculus of Functions of Several Variables[Academic Publishers,India]                     |
| 2. W. Rudin – <i>Principles of Mathematical Analysis</i>  |
| 3. J. Marsden & A. Tromba – <i>Vector Calculus</i> (level surfaces, gradients, Lagrange multipliers)                |
| <b>Web Resources</b>  |
| 1. <a href="https://ocw.mit.edu/">https://ocw.mit.edu/</a>  |
| <b>Evaluation</b> : Theory CIA: 20+5+5=30 Semester Exam: 70   |
| <b>Paper Structure for Theory Semester Exam Module</b> : 7 questions each carrying 10 marks out of 12/13 questions. |

### Course outcomes (COs) and Cognitive Level Mapping

| COs | CO Description  | Cognitive levels |
|-----|---|------------------|
| CO1 | This course on Multivariate Calculus helps in understanding and applying the concepts of limit, continuity and differentiability for functions of more than one variable;   | K2,K3            |
| CO2 | Recognize and interpret the derivative as a linear map and identify Jacobian as the derivative. Grasp the geometric role of the gradient of a function and its interpretation as a normal to the level surfaces.  | K2,K3,K4         |
| CO3 | Differentiability of vector valued functions, examples, linear maps , bilinear maps, differentiability of standard maps, Jacobian matrix and chain rule .Application and properties of Jacobians  | K3               |
| CO4 | Mean value theorem of scalar valued maps on vector domains, functions of vanishing partial derivatives on connected domains. Inverse function theorem, Implicit Function theorem and their applications.  | K3,K4            |
| CO5 | Critical points, Extreme values, Lagrange’s multiplier method, local expression of a function near non-degenerate critical points . Critical points, Extreme values, Lagrange’s multiplier method, local expression of a function near non-degenerate critical points | K3               |
| CO6 | Introduction to double integrals and applications such as computing area and volume.  | K3,K5            |