Syllabus template

Semester: 6	
Programme : Mathematics	
Course : Algebra-4	
Paper code: C3MT230611T	**Credits:4**
Hours/week : 4	
Category: Core/MDC/SEC/VAC : Core	
Theory / Practical / Composite : Theory	
No of Modules : Nil	

Course Overview: Algebra-4

This course develops advanced tools in linear algebra to analyse matrices and linear operators through rank, duality, eigen structure, and inner products. Topics include linear transformations and the rank–nullity theorem, row and column spaces, and different notions of rank; dual spaces, dual bases, transpose maps, and annihilators; eigenspaces, diagonalizability, invariant subspaces, the Cayley–Hamilton theorem, minimal polynomials, and canonical forms. The course also covers inner product spaces and norms, orthogonality, the Gram–Schmidt process, orthogonal complements, and Bessel's inequality, culminating in the study of adjoint, normal, and self-adjoint operators and their diagonalizability.

Course Outcome: On successful completion of the course a student will be able to do the following:

1. **Explain** fundamental concepts of linear transformations, ranks, dual spaces, eigen structure, inner products, and adjoint/normal/self-adjoint operators, using appropriate mathematical language.
2. **Apply** the rank–nullity theorem, duality, eigenvalue/eigenvector computations, and inner product techniques (including Gram–Schmidt) to solve routine problems involving matrices and linear operators.
3. **Analyse** the structure of a linear operator via its row/column spaces, invariant subspaces, minimal polynomial, and canonical forms to determine properties such as rank, diagonalizability, and spectral behaviour.
4. **Evaluate** when an operator is diagonalizable or normal/self-adjoint by using eigen structure, minimal polynomial, and inner product tools, and justify conclusions with rigorous arguments.
5. **Create** coherent multi-step solutions and proofs that integrate rank–nullity, Cayley–Hamilton, minimal polynomials, canonical forms, and inner product methods to address unfamiliar or multi-faceted linear algebra problems.

Prerequisites:

SYLLABUS

UNIT/Module	**CONTENT**	**NUMBER OF CLASSES**	**CO Mapping**	**COGNITIVE LEVEL**
I.	Solving problems of matrices by the use of linear transformations, the rank-nullity theorem. Row space and column space of a matrix. Row rank, column rank, determinant rank and	**32**	**CO1, CO2, CO3, CO4**	**K1, K2, K3, K4**

	their equality. Rank of product of two matrices. Dual spaces, dual basis, double dual , transpose of a linear transformation and its matrix in the dual basis, annihilators. Eigen spaces of a linear operator, diagonalizability, invariant subspaces and Cayley-Hamilton theorem, the minimal polynomial for a linear operator, diagonalizability in connection with minimal polynomial, and canonical forms.			
II.	Inner product spaces and norms-Examples, Cauchy-Schwarz Inequality, Orthogonal and orthonormal basis, Gram-Schmidt orthogonalization process, orthogonal complements, Bessel's inequality . The adjoint of a linear operator. Normal and self-adjoint operators and their diagonalizability.	**20**	**CO5, CO6**	**K4, K5**

Text Books
1. Introduction to Linear Algebra: Gilbert Strang.
2. Linear Algebra--a Geometric Approach: S. Kumaresan.
3. Higher Algebra (Linear and Abstract): S.K.Mapa.
Suggested readings
1. Linear Algebra: Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence.
2. Linear Algebra: Kenneth Hoffman, Ray Kunze.
3. Linear Algebra Done Right: Sheldon Axler.
Web Resources
Evaluation : Theory CIA: 20+5+5=**30** Semester Exam: **70**
Paper Structure for Theory Semester Exam Module: 7 questions each of 10 marks out of a set of **12/13** questions.

Course outcomes (COs) and Cognitive Level Mapping

COs	**CO Description**	**Cognitive levels**

CO1	**Explain** fundamental concepts of linear transformations, ranks, dual spaces, eigen structure, inner products, and adjoint/normal/self-adjoint operators, using appropriate mathematical language.	**K1**
CO2	**Apply** the rank–nullity theorem, duality, eigenvalue/eigenvector computations, and inner product techniques (including Gram–Schmidt) to solve routine problems involving matrices and linear operators.	**K2**
CO3	**Analyse** the structure of a linear operator via its row/column spaces, invariant subspaces, minimal polynomial, and canonical forms to determine properties such as rank, diagonalizability, and spectral behaviour.	**K3**
CO4	**Evaluate** when an operator is diagonalizable or normal/self-adjoint by using eigen structure, minimal polynomial, and inner product tools, and justify conclusions with rigorous arguments.	**K4**
CO5	**Create** coherent multi-step solutions and proofs that integrate rank–nullity, Cayley–Hamilton, minimal polynomials, canonical forms, and inner product methods to address unfamiliar or multi-faceted linear algebra problems.	**K5**