Syllabus template

Semester: 6	
Programme : Mathematics	
Course : Analysis-5	
Paper code: C3MT230621T	**Credits:4**
Hours/week : 4	
Category: Core/MDC/SEC/VAC : Core	
Theory / Practical / Composite : Theory	
No of Modules : Nil	

Course Overview: Analysis-5

This course on Vector Calculus and Power Series offers an in-depth exploration of the differential and integral calculus of vector-valued functions, culminating in the study of power series, Fourier series, and their analytical and physical applications. It blends geometric intuition with rigorous analytical techniques, forming a crucial bridge between calculus, differential equations, and mathematical physics**.** This course provides a comprehensive study of the theory and applications of power series and Fourier series, emphasizing convergence criteria, functional properties, and analytic constructions of fundamental functions. It is designed to solidify understanding of infinite series and their role in analysis, preparing students for advanced courses in real and complex analysis.

Course Outcome: On successful completion of the course a student will be able to do the following:

1. **Explain** the concepts of vector-valued functions, scalar and vector fields, and identify relations among gradient, divergence, curl, and Laplacian.
2. **Compute** derivatives and integrals of vector-valued functions, determine arc lengths of parametrized curves, and interpret physical quantities such as velocity, acceleration, and work.
3. **Analyze** vector integration over curves, surfaces, and volumes, and **verify** fundamental theorems of vector calculus such as Green's, Gauss's, and Stokes' theorems in appropriate physical or geometrical contexts.
4. **Evaluate** conditions for the existence of conservative and irrotational vector fields and **interpret** their physical significance in gravitational and electromagnetic systems.
5. **Determine** the radius and interval of convergence of power series using the Cauchy–Hadamard theorem, and **examine** their uniform and absolute convergence properties.
6. **Construct** exponential, logarithmic, and trigonometric functions using power-series definitions and deduce their key analytical properties.
7. **Apply** Weierstrass approximation and Fourier series techniques to represent continuous and periodic functions, and interpret the implications of Dirichlet's theorem.

Prerequisites:

SYLLABUS

UNIT/Module	**CONTENT**	**NUMBER OF CLASSES**	**CO Mapping**	**COGNITIVE LEVEL**
I. Vector Calculus	Differentiation of vector-valued functions of one scalar variable, velocity, speed, acceleration,	**38**	**CO1, CO2, CO3, CO4**	**K2, K3, K4, K5**

	parametrized curve, unit speed curve, reparameterization, length of a piecewise smooth curve . Implication of the results $\vec{u}.d\vec{u} = 0$ and $\vec{u} \times d\vec{u} = \vec{0}$. Concepts of scalar and vector fields, physical significance. Gradient, Divergence and Curl, Laplacian, Statement and proofs of related vector identities. Vector integration: Line integrals of vector fields along piecewise smooth curves, Application of line integrals, Mass and Work. Fundamental Theorem for line integrals, Conservative vector fields, examples: Gravitational and Electromagnetic fields, Independence of path. Circulation, Irrotational vector fields, and Conservative vector fields are irrotational. In a starlike/simply-connected domain, irrotational vector fields are conservative. Double integrals: Double integration over a rectangular region, double integration over a non-rectangular region, Polar co-ordinates, Surface area and volume, Green's Theorem, area of a plane region bounded by a parametric curve. Integration of a vector field over a plane region. Integration of vector fields over a surface of various form, Stokes' theorem and their applications. Triple integrals, Triple integral over a parallelepiped and solid regions.			

	Volume by triple integrals, cylindrical and spherical co-ordinates. Change of variables in double integrals and triple integrals. The Gauss's divergence theorem.			
II. Power Series	Fundamental theorem of Power series, Cauchy-Hadamard theorem, Determination of radius of convergence. Uniform and absolute convergence of Power series, Properties of sum function. Abel's limit theorems, Uniqueness of power series having the same sum function, Exponential, Logarithmic and trigonometric functions defined by Power-Series and deduction of their salient properties. Weierstrass approximation theorem, Fourier Series, Dirichlet's theorem (statement only) and applications.	**14**	**CO5, CO6, CO7**	**K3, K4, K5, K6**

Text Books
1. Vector Calculus: J. Marsden, A. Tromba.
2. Introduction to Real Analysis: R.G. Bartle D.R. Sherbert, 3rd Ed., John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002.
3. Elementary Analysis: The Theory of Calculus; K.A. Ross, Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.
Suggested readings
1. Basic Multivariate Calculus: A. Weinstein, J. Marsden, A. Tromba.
2. Multivariate Calculus and Geometry: Sean Dineen.
3. Elements of Real Analysis: Charles G. Denlinger, Jones & Bartlett (Student Edition), 2011
Web Resources
Evaluation : Theory CIA: 20+5+5=**30** Semester Exam: **70**
Paper Structure for Theory Semester Exam Module: 7 questions each of 10 marks out of a set of **12/13** questions.

Course outcomes (COs) and Cognitive Level Mapping

COs	CO Description	Cognitive levels
CO1	**Explain** the concepts of vector-valued functions, scalar and vector fields, and identify relations among gradient, divergence, curl, and Laplacian.	**K2**
CO2	**Compute** derivatives and integrals of vector-valued functions, determine arc lengths of parametrized curves,	**K3**

	and interpret physical quantities such as velocity, acceleration, and work.	
CO3	**Analyze** vector integration over curves, surfaces, and volumes, and **verify** fundamental theorems of vector calculus such as Green's, Gauss's, and Stokes' theorems in appropriate physical or geometrical contexts.	**K3, K4**
CO4	**Evaluate** conditions for the existence of conservative and irrotational vector fields and **interpret** their physical significance in gravitational and electromagnetic systems.	**K5**
CO5	**Determine** the radius and interval of convergence of power series using the Cauchy–Hadamard theorem, and **examine** their uniform and absolute convergence properties.	**K4**
CO6	**Construct** exponential, logarithmic, and trigonometric functions using power-series definitions and deduce their key analytical properties.	**K3, K6**
CO7	**Apply** Weierstrass approximation and Fourier series techniques to represent continuous and periodic functions, and interpret the implications of Dirichlet's theorem.	**K3, K5**