Syllabus template

Semester: 6	
Programme : Mathematics	
Course : Differential Equations-2	
Paper code: C3MT230631T	**Credits:4**
Hours/week : 4	
Category: Core/MDC/SEC/VAC : Core	
Theory / Practical / Composite : Theory	
No of Modules : Nil	

Course Overview: Differential Equations-2

This course introduces the theory and applications of continuous dynamical systems and differential equations. Topics include autonomous and non-autonomous systems, critical points, phase planes, trajectories, and local stability analysis using the trace–determinant diagram. Applications in mathematical biology cover population models (Malthusian, logistic, Allee effect), epidemic models (SI, SIS, SIR), and predator–prey systems (Lotka–Volterra, competition, and Richardson's battle model). The differential equations component includes Sturm–Liouville problems, exactness and integrability of total differentials, and power or Frobenius series solutions near singular points. The partial differential equations section covers classification, canonical forms, the method of characteristics, separation of variables, existence and uniqueness results, and the Cauchy–Kovalevskaya theorem. Students will gain modelling skills and analytical techniques to interpret system dynamics in mathematics, biology, and applied sciences.

Course Outcome: On successful completion of the course a student will be able to do the following:

1. **Explain** fundamental concepts of continuous dynamical systems, autonomous and non-autonomous behaviour, and stability of equilibria.
2. **Interpret** and construct phase portraits, phase trajectories, and trace–determinant diagrams to visualize system dynamics.
3. **Analyze** the stability and classification of critical points in one- and two-dimensional systems.
4. **Formulate and evaluate** mathematical models of biological systems, including population dynamics, epidemic spread, and predator–prey interactions.
5. **Solve** ordinary and partial differential equations using analytical techniques such as Sturm–Liouville theory, power series, Frobenius methods, and the method of characteristics.
6. **Evaluate** conditions for exactness, integrability, and uniqueness of solutions in ODEs and PDEs, referencing key theorems like Cauchy–Kovalevskaya.
7. **Develop and interpret** models that describe real-world phenomena across physical and biological contexts using differential equation frameworks.

Prerequisites:

SYLLABUS

UNIT/Module	CONTENT	NUMBER OF CLASSES	CO Mapping	COGNITIVE LEVEL
I. Dynamical Systems	Introduction to Continuous Dynamical Systems, Autonomous and	**24**	**CO1, CO2, CO3, CO4**	**K2, K3, K4, K5**

	Non-Autonomous Systems. Critical points and their classification. Interpretation of the phase plane. Phase trajectory, Phase space and its Interpretation, phase diagram in one variable, idea of local equilibrium, Source, and sink. Local Stability Analysis. Critical points in 2 dimension and their classification. Stability Analysis. Trace Det Diagram. Introduction to Mathematical Biology: Application to single-species population growth model: Malthusian, Verhulst-Pearl Model, Allee effect. Epidemic models like the SI, SIS, and SIR models of Kermack and McKendrick, with an introduction to COVID-19. Lotka-Volterra PreyPredator model without competition and with competition, Battle model of Richardson.			
II. ODE	Sturm-Liouville problems. Total differential equations: conditions of integrability, exactness and techniques of solving them. Power series solution at an ordinary point and Frobenius series solution at a regular singular point.	**14**	**CO5**	**K3**
III. PDE	First Order PDE Classification, Construction and Geometrical Interpretation.	**14**	**CO6, CO7**	**K5, K6**

	Method of Characteristics for obtaining a general Solution of quasi-linear equations. Canonical Forms of First-order Linear Equations. Method of Separation of Variables for solving first-order partial differential equations. Second order PDE: Classifications, Transformation of PDE to Canonical form, Characteristics Equation and Characteristics curve. Study of Initial value problem and Cauchy problems and their Existence and uniqueness of solutions, Cauchy-Kovalevskaya theorem, Hadamard rule for well-posed problems. Superposition principle.			
Text Books				
1. An Introduction to Dynamical Systems and Chaos: G.C.Layek.				
2. An elementary Course in Partial differential Equations: T.Amarnath.				
3. . Linear Partial Differential Equations for Scientists and Engineers: Tyn Myint U and Lokenath Debnath.				
Suggested readings				
1. Differential Equations and Boundary Value Problems: Edwards and Penney.				
2. Differential Equations, Dynamical Systems, and an Introduction to Chaos: Morris W. Hirsch, Robert L. Devaney, and Stephen Smale.				
Web Resources				
Evaluation: Theory CIA: 20+5+5=**30** Semester Exam: **70**				
Paper Structure for Theory Semester Exam Module: 7 questions each of 10 marks out of a set of **12/13** questions.				

Course outcomes (COs) and Cognitive Level Mapping

COs	CO Description	Cognitive levels
CO1	**Explain** fundamental concepts of continuous dynamical systems, autonomous and non-autonomous behaviour, and stability of equilibria.	**K2**

CO2	**Interpret** and construct phase portraits, phase trajectories, and trace–determinant diagrams to visualize system dynamics.	**K3**
CO3	**Analyze** the stability and classification of critical points in one- and two-dimensional systems.	**K4**
CO4	**Formulate and evaluate** mathematical models of biological systems, including population dynamics, epidemic spread, and predator–prey interactions.	**K5**
CO5	**Solve** ordinary and partial differential equations using analytical techniques such as Sturm–Liouville theory, power series, Frobenius methods, and the method of characteristics.	**K3**
CO6	**Evaluate** conditions for exactness, integrability, and uniqueness of solutions in ODEs and PDEs, referencing key theorems like Cauchy–Kovalevskaya.	**K5**
CO7	**Develop and interpret** models that describe real-world phenomena across physical and biological contexts using differential equation frameworks.	**K6**