Syllabus template

Semester: 7	
Programme : Mathematics	
Course : Topology	
Paper code: C4MT230721T	**Credits:4**
Hours/week : 4	
Category: Core/MDC/SEC/VAC : Major	
Theory / Practical / Composite : Theory	
No of Modules : Nil	

Course Overview: Topology

This course introduces the core ideas of topology, evolving from metric spaces to abstract topological structures. It develops fundamental concepts such as open and closed sets, bases, neighborhoods, closure, and continuity, along with constructions like subspaces and product spaces. The course examines convergence, countability axioms, and separation properties, highlighting their roles in distinguishing different classes of topological spaces. Key results involving compactness and connectedness are studied, including Heine–Borel theorem, finite intersection property, and path connectedness. Emphasis is placed on continuous functions, homeomorphisms, and topological equivalence, supported by standard examples in $\mathbb{R},\mathbb{R}^2,\mathbb{R}^3$. The course builds a strong theoretical foundation for advanced studies in analysis and geometry.

Course Outcome: On successful completion of the course a student will be able to do the following:

1. **Explain** fundamental concepts of topology, including open and closed sets, basis, neighborhoods, closure, and interior in abstract topological spaces.
2. **Analyze** different types of topological spaces through subspaces, product spaces, and Kuratowski closure axioms.
3. **Examine** continuity, homeomorphisms, and topological equivalence, and construct continuous functions between spaces.
4. **Evaluate** convergence of sequences in topological spaces and interpret the role of Hausdorff and countability conditions.
5. **Analyze** separation axioms (T_0–T_5) and distinguish between regular, normal, and metric spaces using key theorems.
6. **Apply** the concepts of compactness and connectedness to solve problems and establish key properties in topological spaces.
7. **Classify** topological spaces using compactness, connectedness, and countability properties, and interpret their interrelationships.

Prerequisites:

SYLLABUS

UNIT/Module	CONTENT	NUMBER OF CLASSES	CO Mapping	COGNITIVE LEVEL
I.	Concept of topology in a metric space: Motivation. Abstract topological space: examples, definition. Concept of basis of	**9 classes**	**CO1, CO2**	**K2, K4**

	topological spaces (3). More examples, comparison of topologies, concept of neighbourhood, interior, exterior and boundary, open sets (defined as members of the topology), closed sets (defined as complement of an open set), closure of a set (3). Subspace, (finite) product space. Closure and interior in subspace and product space. Kuratowskii's closure axiom (3).			
II.	Concept of continuous function in topological spaces. Example, definition, construction of continuous functions. Pasting Lemma. Topological equivalence, Homeomorphs, homeomorphic subsets of R,R2,R3 etc. (special mention on known geometric figures) (7). Convergence of sequence in a topological space. Non-uniqueness of limits, limit of a convergent sequence is unique in a Hausdorff space. First countability in context of convergence of a sequence and limit point. Heine's continuity criterion. Properties of a continuous function in connection with Hausdorff space(7). Second countability, separability, Lindeloff properties and their interrelation. They are topological properties but some of them are hereditary and some are productive. Separable metric space is second countable. Separation axioms: T0,T1,T2,T3,T3.5,T4,T5 spaces. Examples. Metric space is T5. Relation between Ti spaces(i=0,1,2,3,4,5). Regular and Normal spaces, Urysohn's lemma, Tietze's extension theorem (without proof). Metric space is normal,	**24 classes**	**CO3, CO4, CO5**	**K3, K4, K5**

	regular Lindeloff space is second countable (10).			
III.	Compactness: concept and examples, definition, [a,b] is compact in R, closed subsets of a compact space is compact. Compact subsets in R. In T2 space compact subsets are closed. Product of two compact space is compact, compact subsets of Rn, Heine-Borel theorem. Lebesgue covering lemma. Compactness is a topological property. Compact sets and continuous functions, FIP. In a compact space a family of closed sets having FIP has non-empty intersection. (8). Concept of connectedness in a topological space, examples, definition, equivalent definition, it is not a hereditary property, intersection of connected sets may not be connected, union of a family of connected sets may not be connected. Closure of a connected set is connected. Connectedness is a topological property. Path connectedness: relation to connectedness. Topologist's sign curve. Examples in R,R2 ,R3 etc. Connected subsets of R. (8) Classification via connectedness and compactness (3).	**19 classes**	**CO6, CO7**	**K3, K4, K5**

Text Books
1. Topology: James R. Munkres
Suggested readings
1. TW Gamelin and RE Greene: Introduction to Topology.
2. G E Bredon: Topology and Geometry.
Web Resources
Evaluation: Theory CIA: **20+5+5=30** Semester Exam: **70**
Paper Structure for Theory Semester Exam Module: 7 questions each of **10** marks out of a set of **13** questions.

Course outcomes (COs) and Cognitive Level Mapping

COs	CO Description	Cognitive levels
CO1	Explain fundamental concepts of topology, including open and closed sets, basis, neighborhoods, closure, and interior in abstract topological spaces.	**K2**
CO2	Analyze different types of topological spaces through subspaces, product spaces, and Kuratowski closure axioms.	**K2, K4**
CO3	Examine continuity, homeomorphisms, and topological equivalence, and construct continuous functions between spaces.	**K3, K4**
CO4	Evaluate convergence of sequences in topological spaces and interpret the role of Hausdorff and countability conditions.	**K4, K5**
CO5	Analyze separation axioms (T_0–T_5) and distinguish between regular, normal, and metric spaces using key theorems.	**K4**
CO6	Apply the concepts of compactness and connectedness to solve problems and establish key properties in topological spaces.	**K3, K4**
CO7	Classify topological spaces using compactness, connectedness, and countability properties, and interpret their interrelationships.	**K4, K5**