Syllabus template

Semester: 7	
Programme: Mathematics	
Course: Algebra-5	
Paper code: C4MT230731T	**Credits:4**
Hours/week: 4	
Category: Core/MDC/SEC/VAC: Major	
Theory / Practical / Composite: Theory	
No of Modules: Nil	

Course Overview: Algebra-5

This course develops core ideas in abstract algebra by integrating ring theory, number theory, and group theory. It begins with divisibility in integral domains, introducing irreducible and prime elements, greatest common divisors, and the structure of unique factorization domains, Euclidean domains, and principal ideal domains, with emphasis on their interrelations. Polynomial rings are studied to establish how algebraic properties extend from a domain to $R[x]$, including unique factorization. The course also explores classical number theoretic results such as Fermat's, Euler's, and Wilson's theorems, along with special topics like the two-square theorem and units in modular arithmetic.

In group theory, the focus shifts to structural concepts including normalizers, centralizers, centers, and subgroup products, followed by external direct products and their applications. The course further develops the theory of automorphisms, including inner automorphisms and automorphism groups of cyclic groups, highlighting the role of factor groups in understanding group structure.

Overall, the course emphasizes algebraic structure, interconnections between different algebraic systems, and problem-solving techniques essential for advanced studies in algebra and number theory.

Course Outcome: On successful completion of the course a student will be able to do the following:

1. **Explain** divisibility in integral domains, including irreducible and prime elements, and compute GCDs in polynomial rings such as $\mathbb{Z}[x]$.
2. **Analyze** algebraic structures such as Euclidean domains, principal ideal domains, and unique factorization domains, and establish their interrelationships with appropriate examples.
3. **Apply** properties of polynomial rings $R[x]$ to determine when they form integral domains, Euclidean domains, or unique factorization domains.
4. **Apply** classical number theoretic results (Fermat's, Euler's, and Wilson's theorems) and analyze properties such as representation as sums of two squares and units in modular arithmetic.
5. **Construct and analyze** external direct products of groups and apply them to solve algebraic problems.

6. **Analyze** automorphisms, inner automorphisms, and automorphism groups of cyclic groups, and apply factor group concepts to study group structure.
Prerequisites:
SYLLABUS

UNIT/Module	**CONTENT**	**NUMBER OF CLASSES**	**CO Mapping**	**COGNITIVE LEVEL**
I.	Unique factorization in Z[x]. Divisibility in integral domains, irreducible, primes **[5]**, GCD, unique factorization domains, Euclidean domains **[5]**. 2-square theorem and properties of $\mathbb{Z}[i]$ and $\mathbb{Z}_p[i]$ **[4].** Every ED is a PID **[2]** **.**Every PID is a UFD **[2].** Examples and problems **[3].** R is an ID implies R[x] is an ID, R is a field implies R[x] is an ED, D is a UFD implies D[x] is a UFD**[4].** Euler's theorem, Fermat's theorem, Wilson's theorem**[3].** Units in $\mathbb{Z}[\sqrt{d}]$ **[2].**	**30**	**CO1, CO2, CO3, CO4**	**K2, K3, K4**
II.	Normalizer, Centralizer, Center of a group, properties and examples **[6].** Product of Subgroups, External direct product of a finite number of groups and their applications, **[4].** Automorphism, inner automorphism, automorphism groups, automorphism groups of finite and infinite cyclic groups, applications of factor groups to automorphism groups **[12].**	**22**	**CO5, CO6, CO7**	**K2, K3, K4, K5**

Text Books
1. Topics in Abstract Algebra: Sen, Ghosh, Mukhopadhyay & Maity.
2. Higher Algebra (Abstract and Linear): S.K. Mapa.
3. Contemporary Abstract Algebra: Joseph Gallian.

Suggested readings
1. Abstract Algebra: Dummit and Foote
2. Topics in Algebra: I.N.Herstein
Web Resources
Evaluation: Theory CIA**: 20+5+5=30** Semester Exam: **70**
Paper Structure for Theory Semester Exam Module: 7 questions each of **10** marks out of a set of **13** questions.

Course outcomes (COs) and Cognitive Level Mapping

COs	**CO Description**	**Cognitive levels**
CO1	Explain divisibility in integral domains, including irreducible and prime elements, and compute GCDs in polynomial rings such as $\mathbb{Z}[x]$..	**K2, K3**
CO2	Analyze algebraic structures such as Euclidean domains, principal ideal domains, and unique factorization domains, and establish their interrelationships with appropriate examples.	**K4**
CO3	Apply properties of polynomial rings $R[x]$ to determine when they form integral domains, Euclidean domains, or unique factorization domains.	**K3, K4**
CO4	Apply classical number theoretic results (Fermat's, Euler's, and Wilson's theorems) and analyze properties such as representation as sums of two squares and units in modular arithmetic.	**K3, K4**
CO5	Examine structural concepts in group theory, including normalizers, centralizers, centers, and subgroup products, with suitable examples.	**K2, K4**
CO6	Construct and analyze external direct products of groups and apply them to solve algebraic problems.	**K3, K4**
CO7	Analyze automorphisms, inner automorphisms, and automorphism groups of cyclic groups, and apply factor group concepts to study group structure.	**K4, K5**