Syllabus template

Semester: 4	
Programme : Mathematics	
Course : Analysis-3	
Paper code: C2MT230411T	**Credits:4**
Hours/week : 4	
Category: Core/MDC/SEC/VAC : Core	
Theory / Practical / Composite : Theory	
No of Modules : Nil	

Course Overview: Analysis-3

This course primarily deals with mainly two cognitive sections, viz, (i) Riemann Integration and Improper Integrals & (ii) Sequences and Series of functions. In the first part the objectives are: Understanding bounded functions, Darboux sums, and conditions for integrability, Riemann integrability and related theorem on integrable functions, studying properties and applications of Riemann integrable functions, Fundamental theorem of integral calculus and mean value theorems (Bonnet & Weierstrass forms), Improper integrals and their convergence tests, Applications of Beta and Gamma functions in evaluating definite integrals. In the second part the objectives are: Point wise and uniform convergence of sequences and series of functions; Cauchy's criterion, Dini's theorem, and Weierstrass M-test related to uniform convergence; continuity, integrability, and differentiability under uniform convergence of sequences &series of functions; Term-by-term integration and differentiation and applications to power series.

Course Outcome:

1. On successful completion of this course, a student will be able to understand and apply the theory of Riemann Integration for bounded real-valued functions defined on closed and bounded intervals along with some of its limitations.
2. The concept of primitives-their existential criteria
3. Study the main types of improper integrals, their convergence criteria and simple applications of Beta and Gamma functions.
4. Grasp the concept of point wise and uniform convergence of sequences & series of functions.
5. Understand how limits under uniform convergence preserve properties such as continuity, integrability, and differentiability. Apply term-by-term integration and differentiation in analyzing sequence and series of functions. One can learn how uniform convergence allows expressing known functions as power series.
6. Differentiation under integral sign containing an arbitrary parameter.

Prerequisites: Basic knowledge about convergence of numerical sequences and series and some common tests related to them. Basic knowledge of Definite integrals taught at 10+2 level.

SYLLABUS

UNIT/Module	**CONTENT**	**NUMBER OF CLASSES**	**CO Mapping**	**COGNITIVE LEVEL**
I.	Recapitulation of Integration by	**8**	**CO1,CO2**	**K1,K2**

	parts. Definition of a set of measure zero. Brief discussion on Step functions and recapitulation of the idea of expressing definite integral as the limit of a sum.Riemann Integration for bounded functions: Partition and refinement of partition of an interval. Upper Darboux sum U(P,f) & Lower Darboux sum L(P,f) and associated results. Upper Riemann integral and Lower Riemann integral .			
II.	Darboux’s theorem. Cauchy’s Necessary and sufficient criterion of Riemann integrability-its equivalence with Darboux’s version. Alternative definition of Riemann integral using tagged partition and equivalence with Darboux’s version. A bounded function on a closed and bounded interval is Riemann integrable if the set of points of discontinuity is a set of measure zero. Problems on Riemann integrability of functions with sets of points of discontinuity having measure zero with special reference to monotone functions, continuous functions,	**16**	**CO1,CO2**	**K4,K5**

	piecewise continuous functions with (i) finite number of points of discontinuities,(ii) infinite number of points of discontinuities having finite number of accumulation points. Integrability of sum, product, quotient, modulus of Riemann integrable functions, Properties of Riemann integrable functions arising from the above results . Function defined by definite integral and its properties. Anti-derivative (indefinite integral).Fundamental theorem of integral calculus .First MVT of integral calculus, second MVT of integral calculus (Bonnet's and Weierstrass' form) and their applications.			
III.	Improper integrals: their convergence, μ-test, comparison test and their applications. Convergence of Beta and Gamma functions and their properties.	**8**	**CO3**	**K2,K3**
IV.	Sequence and Series of functions (defined on a subset of R): Point wise and uniform convergence. Cauchy criterion of uniform convergence.Idea of compact sets in R, Dini's theorem on uniform	**8**	**CO4,CO5**	**K1,K2**

	convergence(statement only). Cauchy's Sup test for uniform convergence of sequences of functions , Weirstrass' M-test for uniform convergence of series (with proof).			
V.	Boundedness. Continuity, Integrability and Differentiability of the limit function of a sequence of functions in case of uniform convergence- the basic difference with pointwise convergence. Statement of Abel's and Dirichlet's test and their applications. Passage to the limit term by term. Sum function: boundedness, continuity, integrability, and differentiability of a series of functions in case of uniform convergence.	**8**	**CO4,CO5**	**K5,K6**
VI.	Differentiation under integral sign containing an arbitrary parameter.	**4**	**CO6**	**K2**

Text Books
1. K.A. Ross, Elementary Analysis: The Theory of Calculus, Springer (SIE).
2. Louis Brand, Advanced Calculus: An Introduction to Classical analysis, Dover.
3. T. M. Apostol, Calculus [Vol I & II]
Suggested readings
1. R.G. Bartle & D.R. Sherbert, Introduction to Real Analysis, 3rd Ed., Wiley,
2. U. Chatterjee, Advanced Mathematical Analysis[Academic Publishers, India]
Web Resources
1. https://en.wikipedia.org/wiki/Riemann_integral

2. https://ddeku.edu.in/Files/2cfa4584-5afe-43ce-aa4b-ad936cc9d3be/Custom/Real%20Analysis(Unit%203,4).pdf
Evaluation : Theory CIA: 20+5+5=**30** Semester Exam: **70**
Paper Structure for Theory Semester Exam Module: 7 questions each of 10 marks out of a set of **12/13** questions.

Course outcomes (COs) and Cognitive Level Mapping

COs	CO Description	Cognitive levels
CO1	On successful completion of this course, a student will be able to understand and apply the theory of Riemann Integration for bounded real-valued functions defined on closed and bounded intervals along with some of its limitations.	**K2,K3**
CO2	The concept of primitives-their existential criteria	**K2,K4**
CO3	Study the main types of improper integrals, their convergence criteria and simple applications of Beta and Gamma functions.	**K2,K3**
CO4	Grasp the concept of point wise and uniform convergence of sequences & series of functions.	**K2,K4**
CO5	Understand how limits under uniform convergence preserve properties such as continuity, integrability, and differentiability. Apply term-by-term integration and differentiation in analyzing sequence and series of functions. Learning how uniform convergence allows expressing known functions as power series.	**K2,K4,K5**
CO6	Differentiation under integral sign containing an arbitrary parameter.	**K2**